Tree-level unitarity constraints in the 2HDM with CP-violation

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Abstract

We obtain tree-level unitarity constraints for the Two Higgs Doublet Model (2HDM) with explicit CP-violation. We limit ourselves to the case with soft violation of the discrete Z_2 symmetry of theory. The key role is played by the rephasing invariance of the 2HDM lagrangian. Our simple approach for derivation of these constraints can be easily transferred to other forms of Higgs sector. We briefly discuss correspondence between possible violation of tree level unitarity limitation and physical content of theory.

The Electroweak Symmetry Breaking in the Standard Model is described usually with the Higgs mechanism. In its simplest variant, an initial Higgs field is an isodoublet of scalar fields with weak isospin $\vec{\sigma}$. The simplest extension of the Higgs sector known as two-Higgs-doublet model (2HDM) consists in introducing two Higgs weak isodoublets of scalar fields ϕ_1 and ϕ_2 with hypercharge $Y = +1$ (for a review, see [1]).

We consider the Higgs potential of 2HDM in the form

$$\begin{aligned} V = \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2\right] \\ -\frac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + \left[m_{12}^2(\phi_1^\dagger\phi_2) + (m_{12}^2)^*(\phi_2^\dagger\phi_1)\right] + m_{22}^2(\phi_2^\dagger\phi_2)\right\}\,. \end{aligned} \tag{1}$$

Here, λ_{1-4}, m_{11}^2 and m_{22}^2 are real (due to hermiticity of the potential), while λ_5 and m_{12} are, in general, complex.

This potential with real coefficients describes the theory without CP violation in the Higgs sector while complex values of some coefficients here make CP violation in Higgs sector possible (a more detailed discussion of many points here and references see in ref. [2]).

• The Higgs–Higgs scattering matrix at high enough energy at tree level contain only s–wave amplitudes; it is described by the quartic part of this potential only. The *tree level unitarity constraints* require that the eigenvalues of this scattering matrix be less than the unitarity limit.

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Since the coefficients of the scattering matrix at high enough energy are given only by parameters λ_i of the Higgs potential, the tree-level unitarity constraints are written as limitations on parameters λ_i (for example, in the minimal SM, with one Higgs doublet and $V = (\lambda/2)(\phi^\dagger\phi - v^2/2)^2$, such unitarity constraint looks as $\lambda < 8\pi$).

Until now, these constraints were considered only for the case without CP violation, i.e. with all coefficients of potential (1) real, [3]. Below we extend these results to the case with explicit CP violation and rederive these constraints in a transparent way, suitable for other extended Higgs sectors discussed in literature.

• The crucial role in the 2HDM is played by the discrete Z_2-symmetry, i.e. symmetry under transformation

$$(\phi_1 \leftrightarrow \phi_1, \phi_2 \leftrightarrow -\phi_2) \quad \text{or} \quad (\phi_1 \leftrightarrow -\phi_1, \phi_2 \leftrightarrow \phi_2). \tag{2}$$

This symmetry forbids (ϕ_1, ϕ_2) mixing.

With this symmetry, the CP violation in the Higgs sector is forbidden and Flavor Changing Neutral Currents (FCNC) are unnatural. In the "realistic" theory this Z_2 symmetry is violated.

Potential (1) contains the m_{12}^2 term, of dimension two, which softly violates the Z_2 symmetry. Soft violation implies that Z_2 symmetry is broken near the mass shell, and is restored at small distances $\ll 1/M_i$, where M_i are masses of Higgs particles[1].

• Potential (1) (and therefore the entire lagrangian) is invariant under the global phase rotations of both doublets $\phi_i \to \phi_i e^{-i\rho_0}$ with common phase ρ_0 (*overall phase freedom*). Besides, the same physical reality (the same set of observables) can be described by a class of lagrangians that differ from each other by independent phase rotation for each doublet [4], [2]

$$\phi_i \to e^{-i\rho_i}\phi_i, \quad \rho_i \text{ real} \quad (i = 1, 2), \quad \rho = \rho_2 - \rho_1\,, \tag{3a}$$

accompanied by compensating phase rotations of parameters of lagrangian:

$$\lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{11(22)}^2 \to m_{11(22)}^2\,, \quad \lambda_5 \to \lambda_5\, e^{-2i\rho}\,, \quad m_{12}^2 \to m_{12}^2 e^{-i\rho}\,. \tag{3b}$$

The invariance of the physical picture in respect to this transformation is called as *the rephasing invariance* and the set of these physically equivalent lagrangians we call as the *rephasing equivalent family.* This one-parametric family is governed by phase difference ρ which we call *the rephasing gauge parameter.* Let us underline that this parameter cannot be determined from any measurement, its choice is only a matter of convenience.

This rephasing invariance is extended to the entire system of fermions and scalars with hard violation of Z_2 symmetry if one supplements transformations (3) by similar transformations for the hard Z_2 symmetry violating terms and Yukawa terms (phases of fermion fields and Yukawa couplings).

• The doublets ϕ_i contain fields with weak isospin projections $\sigma_z = \pm 1/2$:

$$\phi_i = \begin{pmatrix} |+1/2\rangle \\ |-1/2\rangle \end{pmatrix} \equiv \begin{pmatrix} \phi_i^+ \\ n_i + \dfrac{v_i}{\sqrt{2}} \end{pmatrix}, \quad n_i = \frac{\eta_i + i\xi_i}{\sqrt{2}} \quad (i = 1,\, 2)\,, \quad \begin{matrix} v_1 = v\cos\beta \\ v_2 = v\sin\beta \end{matrix}\,. \tag{4}$$

[1] Some authors consider the "most general" Higgs potential with also operators of dimension four, $\left(\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right)(\phi_1^\dagger\phi_2) + h.c.$ which describe hard violation of Z_2 symmetry at all distances. Unfortunately, these discussions are incomplete since these potential terms should be supplemented *for renormalizability* by the mixed kinetic term like $\varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2) + h.c.$ (for more detailed comments see [2]). We do not consider this case.

Here $v_i \equiv \langle\phi_i\rangle$ are the vacuum expectation values (v.e.v.'s) of ϕ_i, which are, in general, complex. For the conjugate fields the isospin projections are $\phi^- = |-1/2\rangle$, $n^* = -|+1/2\rangle$.

Adjusting the global phase, one can make one of these v.e.v.'s (e.g. v_1) real. The rephasing transformation mixes η_i and ξ_i and change phase of v_2.

Two complex isodoublet fields have eight degrees of freedom. Three of them correspond to Goldstone fields $\phi_1^\pm \cos\beta + \phi_2^\pm \sin\beta$, $\xi_1 \cos\beta + \xi_2 \sin\beta$ (which are transformed to longitudinal components of gauge bosons W_L, Z_L). The combinations $H^\pm = \phi_2^\pm \cos\beta - \phi_1^\pm \sin\beta$ describe the observable charged Higgs bosons. The scalar η_1, η_2 and pseudoscalar fields $A = \xi_2 \cos\beta - \xi_1 \sin\beta$ mix to the observable neutral Higgs bosons h_1, h_2, h_3 (which might have no definite CP parity).

• A natural way for derivation of the tree level unitarity constraints is to construct the scattering matrix for all the physical Higgs–Higgs (as well as $Z_L h_i$, $W_L W_L$, etc.) states in the tree approximation at high enough energy (where threshold effects are inessential) and diagonalize it. This very way was realized in the first derivation of such constraint in the Minimal Standard Model [5].

The tree-level unitarity constraints are written for the scattering matrix as the limitations for its eigenvalues. They can be obtained in any basis related to the physical basis by a unitarity transformation. For the considered problem, derivation simplifies in the basis of the non–physical Higgs fields $\phi_i^\pm$, η_i, ξ_i, [3, 6]. The calculations of the mentioned works were limited to the case of CP–conserving Higgs sector, with all λ_i real.

The following simple observation allows us to extend these results to a more general case with CP violation. Let us repeat that the unitarity constraints are written for the very high energy Higgs–Higgs scattering matrix, which is expressed via quartic terms of potential λ_i only. *Starting from representation with complex λ_5, one can perform rephasing transformation (3)* (with $\rho = -arg(\lambda_5)/2$) *to obtain the real parameter λ_{5r} and derive unitarity constraints just as in the CP-conserving case. Since the physical picture is invariant under such rephasing transformation, the results with $\lambda_{5r} = |\lambda_5|$ correspond also to the initial situation*[2].

• With our choice of rephasing representation with real λ_5, the CP is conserved at very high energy, so it is sufficient to consider complex neutral fields n_i (4) instead of separate scalars η_i and pseudoscalars ξ_i (which would mix in an arbitrary rephasing gauge). In the the high-energy scalar-scalar scattering, the total weak isospin $\vec{\sigma}$, the total hypercharge Y are conserved. Besides, since the quartic part of the Higgs potential (1) conserves Z_2 symmetry, the Higgs–Higgs states can also be classified according to their value of the Z_2-parity: $\phi_i\phi_i$, $\phi_i\phi_i^*$, etc. will be called the Z_2-even states, and $\phi_1\phi_2$, $\phi_1\phi_2^*$, etc. will be called the Z_2-odd states, and this Z_2 parity is also conserved at very high energy.

Table 1 shows the classification of the two-scalar states constructed from fields $\phi_i^\pm$, n_i and n_i^* according to the Z_2 parity, hypercharge, weak isospin and its z-projection. Only the transitions between the states within each cell of Table 1 are possible[3].

[2] This rephasing gauge is different from that which is useful for describing CP-violation [2].

[3] The double-charged states with $Y = 2$, $\sigma = \sigma_z = 1$ were omitted in the analysis of [3].

		$Y=2$		$Y=0$	
σ	σ_z	Z_2 even	Z_2 odd	Z_2 even	Z_2 odd
1	1	$\begin{pmatrix} \phi_1^+\phi_1^+ \\ \phi_2^+\phi_2^+ \end{pmatrix}$	$\phi_1^+\phi_2^+$	$\begin{pmatrix} \phi_1^+ n_1^* \\ \phi_2^+ n_2^* \end{pmatrix}$	$\begin{pmatrix} \phi_1^+ n_2^* \\ \phi_2^+ n_1^* \end{pmatrix}$
	0	$\begin{pmatrix} \phi_1^+ n_1 \\ \phi_2^+ n_2 \end{pmatrix}$	$\dfrac{\phi_1^+ n_2 + \phi_2^+ n_1}{\sqrt{2}}$	$\begin{pmatrix} \dfrac{\phi_1^+\phi_1^- - n_1 n_1^*}{\sqrt{2}} \\ \dfrac{\phi_2^+\phi_2^- - n_2 n_2^*}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \dfrac{\phi_1^+\phi_2^- - n_1 n_2^*}{\sqrt{2}} \\ \dfrac{\phi_2^+\phi_1^- - n_2 n_1^*}{\sqrt{2}} \end{pmatrix}$
	-1	$\begin{pmatrix} n_1 n_1 \\ n_2 n_2 \end{pmatrix}$	$n_1 n_2$	$\begin{pmatrix} \phi_1^- n_1 \\ \phi_2^- n_2 \end{pmatrix}$	$\begin{pmatrix} \phi_1^- n_2 \\ \phi_2^- n_1 \end{pmatrix}$
0	0	absent	$\dfrac{\phi_1^+ n_2 - \phi_2^+ n_1}{\sqrt{2}}$	$\begin{pmatrix} \dfrac{\phi_1^+\phi_1^- + n_1 n_1^*}{\sqrt{2}} \\ \dfrac{\phi_2^+\phi_2^- + n_2 n_2^*}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \dfrac{\phi_1^+\phi_2^- + n_1 n_2^*}{\sqrt{2}} \\ \dfrac{\phi_2^+\phi_1^- + n_2 n_1^*}{\sqrt{2}} \end{pmatrix}$

Table 1. The two-Higgs states with different quantum numbers.

The states with $Y=-2$, $\sigma=1$ are obtained from those for $Y=2$ by charge conjugation and change of sign of σ_z. For $Y=\pm 2$, the Z_2 even state with $\sigma=0$ cannot occur due to Bose-Einstein symmetry of identical scalars.

The scattering matrices for different states are determined completely by values of the hypercharge, the total weak isospin and the Z_2 parity of the initial Higgs–Higgs state, they are independent of σ_z. The scattering matrices for each set of these quantum numbers are listed in Table 2, their coefficients are easily seen from the potential (1).

Y	σ	Z_2 even	Z_2 odd
± 2	1	$\begin{pmatrix} \lambda_1 & \lambda_{5r} \\ \lambda_{5r} & \lambda_2 \end{pmatrix}$	$\lambda_3+\lambda_4$
± 2	0	–	$\lambda_3-\lambda_4$
0	1	$\begin{pmatrix} \lambda_1 & \lambda_4 \\ \lambda_4 & \lambda_2 \end{pmatrix}$	$\begin{pmatrix} \lambda_3 & \lambda_{5r} \\ \lambda_{5r} & \lambda_3 \end{pmatrix}$
0	0	$\begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 \\ 2\lambda_3+\lambda_4 & 3\lambda_2 \end{pmatrix}$	$\begin{pmatrix} \lambda_3+2\lambda_4 & 3\lambda_{5r} \\ 3\lambda_{5r} & \lambda_3+2\lambda_4 \end{pmatrix}$

Table 2. Scattering matrices for different Higgs–Higgs states (with factor $1/(8\pi)$).

The case $Y=0$, $\sigma_z=0$ demands special discussion. Let us consider, for example, term $V_1=\lambda_1(\phi_1^\dagger\phi_1)^2/2$ in the potential. In terms of operators $\hat{\phi}^\pm$, $\hat{n}$ (omitting subscript 1 for brevity), we have $\hat{V}_1=\lambda_1(\hat{\phi}^-\hat{\phi}^+ + \hat{n}\hat{n}^*)^2/2$. Within the subspace of neutral two-particle states with $Y=0$, one can rewrite $\hat{V}_1$ after simple combinatorics as

$$\hat{V}_1=\frac{\lambda_1}{2}\left[4|\phi^+\phi^-\rangle\langle\phi^+\phi^-|+2|\phi^+\phi^-\rangle\langle nn^*|+2|nn^*\rangle\langle\phi^+\phi^-|+4|nn^*\rangle\langle nn^*|\right]$$

$$=3\lambda_1\frac{|\phi^+\phi^-\rangle+|nn^*\rangle}{\sqrt{2}}\cdot\frac{\langle\phi^+\phi^-|+\langle nn^*|}{\sqrt{2}}+\lambda_1\frac{|\phi^+\phi^-\rangle-|nn^*\rangle}{\sqrt{2}}\cdot\frac{\langle\phi^+\phi^-|-\langle nn^*|}{\sqrt{2}}\,.$$

One can see precisely these numerical coefficients in Table 2.

The classification scheme based on the quantum numbers of EW theory σ, σ_z, Y, and Z_2 looks more *natural* for the considered problems than both the $O(4)$-classification (in the minimal SM) introduced in [5] and the scheme based on new quantum numbers C, G, and Y_π introduced in [6].

The scheme proposed above can be readily exploited in the study of some other multi-Higgs sectors. For example, the analysis in the cases of widely discussed 2 doublet + singlet model or doublet–triplet model should be also very simple, the analysis of three-doublet Higgs model, etc, is expected to be not very difficult.

Note that $\lambda_{5r} \equiv |\lambda_5|$. Therefore, one can present all eigenvalues of these scattering matrices $\Lambda^{Z_2}_{Y\sigma\pm}$ even for complex values of λ_5 as

$$\begin{aligned}
\Lambda^{even}_{21\pm} &= \frac{1}{2}\left(\lambda_1+\lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2+4|\lambda_5|^2}\right), \quad \Lambda^{odd}_{21}=\lambda_3+\lambda_4\,, \quad \Lambda^{odd}_{20}=\lambda_3-\lambda_4\,,\\
\Lambda^{even}_{01\pm} &= \frac{1}{2}\left(\lambda_1+\lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2+4\lambda_4^2}\right), \quad \Lambda^{odd}_{01\pm}=\lambda_3\pm|\lambda_5|\,, \qquad (5)\\
\Lambda^{even}_{00\pm} &= \frac{1}{2}\left[3(\lambda_1+\lambda_2) \pm \sqrt{9(\lambda_1-\lambda_2)^2+4(2\lambda_3+\lambda_4)^2}\right), \quad \Lambda^{odd}_{00\pm}=\lambda_3+2\lambda_4\pm 3|\lambda_5|\,.
\end{aligned}$$

The tree-level unitarity constraints can be written now as

$$|\Lambda^{Z_2}_{Y\sigma\pm}| < 8\pi\,. \qquad (6)$$

They differ from those obtained in ref. [3] only by the change $\lambda_5 \to |\lambda_5|$.

• Note that in the considered tree approximation the masses of Higgs bosons are composed from quantities λ_i and quantity $\nu \propto Re(\overline{m}^2_{12})/v_1v_2$ (the quantity m^2_{12} in a special rephasing gauge different from that used above – see [2] and [7] for details). Since parameter m^2_{12} does not enter the quartic interactions, the above unitarity constraints, generally, do not set any limitation on masses of observable Higgs bosons, which was explicitly noted in [6]. Reasonable limitations on these masses can be obtained for some specific values of ν. For example, for reasonably small value of ν one can have the lightest Higgs boson mass of about 120 GeV and the masses of other Higgs bosons can be up to about 600 GeV without violation of tree-level unitarity [2]. At large ν, masses of all Higgs bosons except the lightest one can be very large without violation of unitarity constraint.

• Let us discuss briefly some new features, which are brought up by the situation with unitarity constraints in 2HDM.

The unitarity constraints were obtained first [5] in the minimal SM. In this model, the Higgs boson mass $M_H = v\sqrt{\lambda}$, and its width Γ (given mainly by decay to longitudinal components of gauge bosons W_L, Z_L) grows as $\Gamma \propto M_H^3$. The unitarity limit corresponds to the case when $\Gamma_H \approx M_H$, so that the physical Higgs boson disappears, the strong interaction in the Higgs sector is realized as strong interaction of longitudinal components of gauge bosons W_L, Z_L at $\sqrt{s} > v\sqrt{\lambda} \gtrsim v\sqrt{8\pi} \approx 1.2$ TeV. Therefore, if λ exceeds the tree-level unitarity limitation, the discussion in terms of observable Higgs particle becomes meaningless, and a new physical picture for the Electroweak Symmetry Breaking in SM arises.

Such type of correspondence among the tree level unitarity limit, realization of the Higgs field as more or less narrow resonance and a possible strong W_LW_L and Z_LZ_L interaction, can generally be violated in the 2HDM if values of λ_i differ from each other essentially. Large number of degrees of freedom of 2HDM generates situations when some of Higgs bosons of this theory are "normal" more or less narrow scalars (whose properties can be estimated perturbatively), while the other scalars and (or) W_L, Z_L interact strongly at sufficiently high energy. It can happen that some of the latter can be realized as physical particles, while the other disappear from particle spectrum like Higgs boson in SM with large λ. In such cases the unitarity constraints work in different way for different *physical* channels. The list of possibilities will be studied elsewhere.

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